

DAY — 08

SEAT NUMBER

--	--	--	--	--	--

2024	VII	25	1100	J-178	(E)
MATHEMATICS & STATISTICS (88) (COMMERCE)					
Time : 3 Hrs.		(12 Pages)		Max. Marks : 80	

General Instructions :

- (i) All questions are compulsory.
- (ii) There are 6 questions divided into two sections.
- (iii) Write answers of Section-I and Section-II in the same answer book.
- (iv) Use of logarithmic tables is allowed. Use of calculator is not allowed.
- (v) For L.P.P. and Time series graph paper is not necessary. Only rough sketch of graph is expected.
- (vi) Start answer to each question on a new page.
- (vii) For each objective type of questions (i.e. Q.1 and Q.4) only the first attempt will be considered for evaluation.

SECTION - I

Q. 1. (A) Select and write the correct answer of the following multiple choice type of questions (1 mark each) : **[12]**

(6)

(i) Which of the following sentences is a statement in logic:

- (a) He is a good actor.
- (b) Did you eat lunch yet?
- (c) Every real number is a complex number.
- (d) Bring the motor car here.

(ii) If $y = 2x^2 + \log 2 + 5$ then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.

- (a) x (b) $4x$
- (c) $2x + \log 2$ (d) $-4x$

(iii) If $x = 2at^2, y = 4at$ then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.

- (a) $-\frac{1}{2at^2}$ (b) $\frac{1}{2at^3}$
- (c) $\frac{1}{t}$ (d) $\frac{1}{4at^3}$

(iv) The equation of tangent to the curve $y = x^2 + 4x + 1$ at $P(-1, -2)$ is $\underline{\hspace{2cm}}$.

- (a) $2x - y = 0$ (b) $x + 2y + 5 = 0$
- (c) $2x + 4 = 3y$ (d) $5x + y = 1$

(v) $\int_{-2}^3 \frac{dx}{x+5} = \underline{\hspace{2cm}}$.

- (a) $-\log\left(\frac{8}{3}\right)$ (b) $3 \log\left(\frac{3}{8}\right)$
- (c) $\log\left(\frac{8}{3}\right)$ (d) $-2 \log\left(\frac{3}{8}\right)$

(vi) The order and degree of the differential equation

$$\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 + 8 = 0 \text{ are } \underline{\hspace{2cm}}.$$

- (a) order = 2, degree = 2
- (b) order = 1, degree = 2
- (c) order = 1, degree = 1
- (d) order = 2, degree = 1

(B) State whether the following statements are true or false
(1 mark each) : (3)

- (i) Every identity matrix is a scalar matrix.
- (ii) The rate of change of demand (x) of a commodity
w.r.t. its price (y) is $\frac{dy}{dx}$.
- (iii) The integrating factor of $\frac{dy}{dx} - y = x$ is e^x .

(C) Fill in the following blanks (1 mark each) : (3)

- (i) If $y = x \cdot \log x$ then $\frac{d^2y}{dx^2} = \underline{\hspace{2cm}}$.
- (ii) If the marginal revenue $R_m = 40$ and elasticity of demand η is 5, then the average revenue R_A will be $\underline{\hspace{2cm}}$.
- (iii) Area of the region bounded by $y = x^4$, $x = 1$, $x = 5$ and the X-axis will be $\underline{\hspace{2cm}}$.

Q. 2. (A) Attempt any TWO of the following questions (3 marks each) : (6) [14]

- (i) Write converse, inverse and contrapositive of the following statement :
If the train reaches on time then I can catch the connecting flight.

(ii) If $A = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix}$ and if $(A+B)^2 = A^2 + B^2$, find value of a and b .

(iii) Find $\frac{dy}{dx}$, if $y = (x)^x + (a)^x$.

(B) Attempt any TWO of the following questions (4 marks each): (8)

(i) Evaluate: $\int \frac{x}{4x^4 - 20x^2 - 3} dx$

(ii) Evaluate: $\int_1^3 \log x dx$

(iii) In a certain culture of bacteria, their rate of increase is proportional to the number present. If it is found that the number doubles in 4 hours, find the number of times the bacteria are increased in 12 hours.

Q. 3. (A) Attempt any TWO of the following questions (3 marks each): (6) [14]

(i) A metal wire of 36 cm length is bent to form a rectangle. Find its dimensions when its area is maximum.

(ii) Evaluate: $\int \frac{2x+1}{x(x-1)(x-4)} dx$

(iii) Find the area of region bounded by $y^2 = 25x$ and the line $x = 4$.

(B) Attempt any ONE of the following questions (4 marks each): (4)

(i) Using the truth table, verify :
 $\sim(p \rightarrow \sim q) \equiv p \wedge \sim(\sim q) \equiv p \wedge q$

(ii) Solve the following equations by method of inversion:
 $x + y + z = 1$, $x - y + z = 2$, $x + y - z = 3$

(C) Attempt any ONE of the following questions (Activity)

(4 marks each):

(4)

- (i) The cost C for producing x articles is given as $C = x^3 - 16x^2 + 47x$. For what values of x the average cost is decreasing?

Solution : Given $C = x^3 - 16x^2 + 47x$

$$\text{Average cost } C_A = \frac{C}{x}$$

$$\therefore C_A = \boxed{}$$

Differentiating w.r.t. x , we get

$$\frac{d}{dx}(C_A) = \boxed{}$$

We know that C_A is decreasing

$$\text{if } \frac{d}{dx}(C_A) \boxed{} 0$$

$$\therefore 2x - 16 < 0$$

$$\therefore 2x < 16$$

$$\therefore x < \boxed{}$$

$\therefore$ Average cost is decreasing for $x \in (0, 8)$

- (ii) Solve the differential equation :

$$y - x \frac{dy}{dx} = 0$$

Solution : Given equation is $y - x \frac{dy}{dx} = 0$

Separating the variables we get

$$\frac{dx}{\boxed{}} = \frac{dy}{\boxed{}}$$

Integrating we get,

$$\int \frac{dx}{\boxed{}} = \int \frac{dy}{\boxed{}} + c$$

$$\therefore \log x = \boxed{} + c$$

$$\therefore \log x - \log y = \log c_1, \text{ Where } c = \log c_1$$

$$\therefore \log\left(\frac{x}{y}\right) = \log c_1$$

$$\therefore \frac{x}{\boxed{}} = c_1$$

Hence the required solution is $x = c_1 y$.

SECTION - II

Q. 4. (A) Select and write the correct answer of the following multiple choice type of questions (1 mark each) : **[12]**

(i) The date on which the period of the bill expires is called ____.

- (a) Legal due date
- (b) Grace date
- (c) Nominal due date
- (d) Date of drawing

(ii) A person insured a property of ₹ 4,00,000. The rate of premium is ₹ 35 per thousand p.a. The amount of annual premium is ____.

- (a) ₹ 14,000
- (b) ₹ 24,000
- (c) ₹ 34,000
- (d) ₹ 15,000

(iii) Paasche's Price Index Number is given by ____.

- (a) $\frac{\sum p_0 q_0}{\sum p_1 q_0} \times 100$
- (b) $\frac{\sum p_0 q_1}{\sum p_1 q_1} \times 100$
- (c) $\frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$
- (d) $\frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$

- (iv) If jobs I, II, III have processing times as 8, 6, 5 on machine M_1 and 8, 3, 4 on machine M_2 in the order M_1 - M_2 . Then the optimal sequence is ____.
- (a) I II III (b) I III II
(c) II I III (d) III II I
- (v) If $E(X) = 4$ and X follows Poisson's distribution then $V(X) =$ ____.
- (a) 2 (b) -2
(c) 4 (d) -4
- (vi) Three coins are tossed simultaneously. X is the number of heads. Then the expected value of X is ____.
- (a) 1 (b) 1.5
(c) 1.9 (d) 1.017

(B) State whether the following statements are true or false (3)
(1 mark each) :

- (i) In the regression of Y on X , X is the independent variable and Y is the dependent variable.
- (ii) The region represented by the inequalities $x \leq 0, y \leq 0$ lies in the first quadrant.
- (iii) In an assignment problem, if the number of columns are greater than number of rows, then a dummy column is added.

(C) Fill in the following blanks (1 mark each) : (3)

- (i) If an agent charges 12% commission on the sales of ₹ 48,000 then his total commission is ₹ ____.
- (ii) The optimal value of the objective function is attained at the ____ points of feasible region.

(iii) Given p.d.f. of a continuous random variable X is,

$$f(x) = \frac{x}{8}, \text{ for } 0 < x < 4$$

$$= 0, \text{ otherwise}$$

then $P(1 < x < 2) = \underline{\hspace{2cm}}$.

Q. 5. (A) Attempt any TWO of the following questions (3 marks each): (6) [14]

- (i) Find the rate of interest compounded annually if an immediate annuity of ₹ 20,000 per year amounts to ₹ 41,000 in 2 years.
- (ii) Find the Value Index Number using Simple Aggregate Method for the following data :

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	30	22	40	18
B	40	16	60	12
C	10	38	15	24
D	50	12	60	16
E	20	28	25	36

- (iii) Five jobs must pass through a lathe and a surface grinder, in that order. The processing times in hours are shown below. Determine the optimal sequence of the jobs. Also find the total elapsed time :

Jobs	I	II	III	IV	V
Lathe	4	1	5	2	5
Surface grinder	3	2	4	3	6

(B) Attempt any TWO of the following questions (4 marks each) : (8)

- (i) A bill was drawn on 14th April for ₹ 7,000 and was discounted on 6th July at 5% p.a. The banker paid ₹ 6,930 for the bills. Find the period of the bill.
- (ii) The following table gives the production of steel (in millions of tons) for years 1976 to 1986 :

Years	1976	1977	1978	1979	1980
Production	0	4	4	2	6

1981	1982	1983	1984	1985	1986
8	5	9	4	10	10

Fit a trend line to the above data by the method of least squares.

- (iii) Solve the following L.P.P. by graphical method :

Maximize : $z = 7x + 11y$

Subject to : $3x + 4y \leq 24$,

$5x + 3y \leq 30$,

$x \geq 0, y \geq 0$

Q. 6. (A) Attempt any TWO of the following questions (3 marks each) : (6) [14]

- (i) For a bivariate data, $\bar{x} = 53$, $\bar{y} = 28$, $b_{yx} = -1.2$,
 $b_{xy} = -0.3$
 Find : (a) Correlation coefficient between x and y .
 (b) Estimate y for $x = 50$.
- (ii) Given that $\Sigma p_0q_0 = 220$, $\Sigma p_0q_1 = 380$, $\Sigma p_1q_1 = 350$ and Marshall-Edgeworth's Price Index Number is 150, find Laspeyre's Price Index Number.

- (iii) The following data gives the production of bleaching powder (in '000 tons) for the years 1962 to 1972 :

Years	1962	1963	1964	1965	1966
Production	0	0	1	1	4

1967	1968	1969	1970	1971	1972
2	4	9	7	10	8

Obtain the trend values for the above data using 5 yearly moving averages.

- (B) Attempt any ONE of the following questions (4 marks each): (4)

- (i) Four new machines M_1 , M_2 , M_3 and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M_2 cannot be placed at C and M_3 cannot be placed at A. The cost matrix is given below :

Machines	Places				
	A	B	C	D	E
M_1	4	6	10	5	6
M_2	7	4	—	5	4
M_3	—	6	9	6	2
M_4	9	3	7	2	3

Find the optimal assignment schedule.

- (ii) There are 10% defective items in a large bulk of items. What is the probability that a sample of 4 items will include not more than one defective item?

(C) Attempt any ONE of the following questions (Activity) (4)
(4 marks each) :

- (i) The equations of the two regression lines are $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$. Obtain the correlation coefficient between x and y .

Solution :

To find correlation coefficient, we have to find the regression coefficients b_{yx} and b_{xy} .

Let $3x + 2y = 26$ be equation of the line of regression of y on x .

This gives $y = \boxed{} x + 13$

$$\therefore b_{yx} = -\frac{3}{2}$$

Now consider $6x + y = 31$ as equation of the line of regression of x on y .

This can be written as $x = \boxed{} y + \frac{31}{6}$

$$\therefore b_{xy} = -\frac{1}{6}$$

Now $r^2 = \boxed{} = 0.25$

As both b_{yx} and b_{xy} are negative,

$$\therefore r = \boxed{}$$

- (ii) The probability distribution of X is as follows :

x	0	1	2	3	4
$P(X = x)$	0.1	k	$2k$	$2k$	k

- Find :
- (a) k
 - (b) $P(X < 2)$
 - (c) $P(1 \leq X < 4)$
 - (d) $F(2)$

Solution :

The table gives a probability distribution.

$$\therefore \sum p_i = 1$$

$$\therefore 0.1 + k + 2k + 2k + k = 1$$

$$(a) \quad k = \boxed{}$$

$$(b) \quad P(X < 2) = P(X = 0) + P(X = 1) \\ = \boxed{}$$

$$(c) \quad P(1 \leq X < 4) = P(1) + P(2) + P(3) \\ = \boxed{}$$

$$(d) \quad F(2) = P(X \leq 2) \\ = P(0) + P(1) + P(2) \\ = \boxed{}$$

