DAY — 09 SEAT NUMBER

2023	III	03	1100	J-262	(E)

MATHEMATICS & STATISTICS (40)
(ARTS & SCIENCE)

Time : 3 Hrs. **(8 Pages)** **Max. Marks : 80**

General instructions :

*The question paper is divided into **FOUR** sections.*

(1) ***Section A :*** *Q. 1 contains **Eight** multiple choice type of questions, each carrying **Two** marks.*
*Q. 2 contains **Four** very short answer type questions, each carrying **one** mark.*

(2) ***Section B :*** *Q. 3 to Q. 14 contain **Twelve** short answer type questions, each carrying **Two** marks. (Attempt any **Eight**)*

(3) ***Section C :*** *Q. 15 to Q. 26 contain **Twelve** short answer type questions, each carrying **Three** marks. (Attempt any **Eight**)*

(4) ***Section D :*** *Q. 27 to Q. 34 contain **Eight** long answer type questions, each carrying **Four** marks. (Attempt any **Five**)*

(5) Use of log table is allowed. Use of calculator is not allowed.

(6) Figures to the right indicate full marks.

(7) Use of graph paper is <u>not</u> necessary. Only rough sketch of graph is expected.

(8) For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabet, e.g., (a) ...../ (b) ..... / (c) ...../ (d)....., etc. No marks shall be given, if <u>ONLY</u> the correct answer or the alphabet of correct answer is written. Only the first attempt will be considered for evaluation.

(9) Start answer to each section on a new page.

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SECTION – A

Q. 1. Select and write the correct answer for the following multiple choice type of questions : [16]

(i) If $p \wedge q$ is F, $p \rightarrow q$ is F then the truth values of p and q are _____ respectively.

(a) T, T (b) T, F

(c) F, T (d) F, F (2)

(ii) In ΔABC, if $c^2 + a^2 - b^2 = ac$, then $\angle B =$ _____.

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$ (2)

(iii) The area of the triangle with vertices (1, 2, 0), (1, 0, 2) and (0, 3, 1) in sq. unit is _____.

(a) $\sqrt{5}$ (b) $\sqrt{7}$

(c) $\sqrt{6}$ (d) $\sqrt{3}$ (2)

(iv) If the corner points of the feasible solution are (0,10), (2, 2) and (4, 0) then the point of minimum $z = 3x + 2y$ is _____.

(a) (2, 2) (b) (0, 10)

(c) (4, 0) (d) (3, 4) (2)

(v) If y is a function of x and $\log(x + y) = 2xy$, then the value of $y'(0) =$ _____.

(a) 2 (b) 0

(c) – 1 (d) 1 (2)

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(vi) $\int \cos^3 x\,dx =$ _____.

(a) $\frac{1}{12}\sin 3x + \frac{3}{4}\sin x + c$

(b) $\frac{1}{12}\sin 3x + \frac{1}{4}\sin x + c$

(c) $\frac{1}{12}\sin 3x - \frac{3}{4}\sin x + c$

(d) $\frac{1}{12}\sin 3x - \frac{1}{4}\sin x + c$ (2)

(vii) The solution of the differential equation $\frac{dx}{dt} = \frac{x \log x}{t}$ is _____

(a) $x = e^{ct}$ (b) $x - e^{ct} = 0$

(c) $x = e^{t} + t$ (d) $x e^{t} = 0$ (2)

(viii) Let the probability mass function (p.m.f.) of a random variable X be $P(X = x) = {}^4C_x \left(\frac{5}{9}\right)^x \times \left(\frac{4}{9}\right)^{4-x}$, for $x = 0, 1, 2, 3, 4$ then $E(X)$ is equal to _____

(a) $\frac{20}{9}$ (b) $\frac{9}{20}$

(c) $\frac{12}{9}$ (d) $\frac{9}{25}$ (2)

Q. 2. Answer the following questions : [4]

(i) Write the joint equation of co-ordinate axes. (1)

(ii) Find the values of c which satisfy $|c\,\bar{u}| = 3$ where $\bar{u} = \hat{i} + 2\hat{j} + 3\hat{k}$. (1)

(iii) Write $\int \cot x\,dx$. (1)

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(iv) Write the degree of the differential equation

$$e^{\frac{dy}{dx}} + \frac{dy}{dx} = x$$ (1)

SECTION – B

Attempt any EIGHT of the following questions : **[16]**

Q. 3. Write inverse and contrapositive of the following statement :
If $x < y$ then $x^2 < y^2$ (2)

Q. 4. If $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is a non singular matrix, then find A^{-1} by elementary row transformations.

Hence write the inverse of $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (2)

Q. 5. Find the cartesian co-ordinates of the point whose polar co-ordinates are $\left(\sqrt{2}, \frac{\pi}{4}\right)$. (2)

Q. 6. If $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines and $h^2 = ab \neq 0$ then find the ratio of their slopes. (2)

Q. 7. If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points A, B, C respectively and $5\bar{a} + 3\bar{b} - 8\bar{c} = \bar{0}$ then find the ratio in which the point C divides the line segment AB. (2)

Q. 8. Solve the following inequations graphically and write the corner points of the feasible region :
$2x + 3y \leq 6,\ x + y \geq 2,\ x \geq 0,\ y \geq 0$ (2)

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Q. 9. Show that the function $f(x) = x^3 + 10x + 7$, $x \in R$ is strictly increasing. (2)

Q. 10. Evaluate : $\int_0^{\frac{\pi}{2}} \sqrt{1 - \cos 4x}\, dx$ (2)

Q. 11. Find the area of the region bounded by the curve $y^2 = 4x$, the X-axis and the lines $x = 1$, $x = 4$ for $y \geq 0$. (2)

Q. 12. Solve the differential equation

$\cos x \cos y\, dy - \sin x \sin y\, dx = 0$ (2)

Q. 13. Find the mean of number randomly selected from 1 to 15. (2)

Q. 14. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$. https://www.maharashtrastudy.com (2)

SECTION – C

Attempt any EIGHT of the following questions : **[24]**

Q. 15. Find the general solution of $\sin\theta + \sin 3\theta + \sin 5\theta = 0$ (3)

Q. 16. If $-1 \leq x \leq 1$, then prove that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ (3)

Q. 17. If θ is the acute angle between the lines represented by

$ax^2 + 2hxy + by^2 = 0$ then prove that $\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$ (3)

Q. 18. Find the direction ratios of a vector perpendicular to the two lines whose direction ratios are $-2, 1, -1$ and $-3, -4, 1$. **(3)**

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Q. 19. Find the shortest distance between lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ and } \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$$ (3)

Q. 20. Lines $\bar{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ and $\bar{r}=(4\hat{i}-3\hat{j}+2\hat{k})$ $+\mu(\hat{i}-2\hat{j}+2\hat{k})$ are coplanar. Find the equation of the plane determined by them. (3)

Q. 21. If $y=\sqrt{\tan x+\sqrt{\tan x+\sqrt{\tan x+.....+\infty}}}$, then

show that $\frac{dy}{dx}=\frac{\sec^2 x}{2y-1}$.

Find $\frac{dy}{dx}$ at $x=0$. (3)

Q. 22. Find the approximate value of $\sin(30^\circ 30')$.

Given that $1^\circ=0.0175^c$ and $\cos 30^\circ=0.866$ (3)

Q. 23. Evaluate $\int x\tan^{-1}x\,dx$ (3)

Q. 24. Find the particular solution of the differential equation

$\frac{dy}{dx}=e^{2y}\cos x$, when $x=\frac{\pi}{6}, y=0$. (3)

Q. 25. For the following probability density function of a random variable X, find (a) $P(X<1)$ and (b) $P(|X|<1)$.

$$f(x)=\frac{x+2}{18} \text{ ; for } -2<x<4$$

$$=0 \quad \text{, otherwise}$$ (3)

Q. 26. A die is thrown 6 times. If 'getting an odd number' is a success, find the probability of at least 5 successes. (3)

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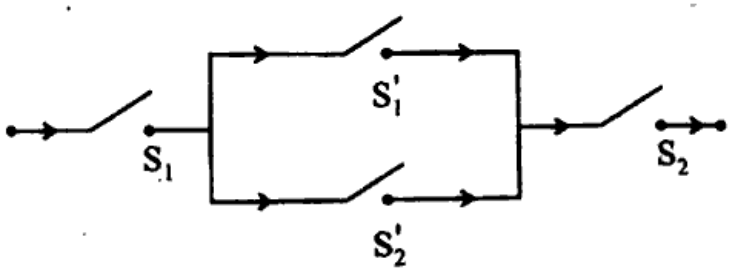
SECTION – D

Attempt any FIVE of the following questions : **[20]**

Q. 27. Simplify the given circuit by writing its logical expression. Also write your conclusion.



(4)

Q. 28. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ verify that

$$A(\text{adj}A) = (\text{adj}A)A = |A|I$$ (4)

Q. 29. Prove that the volume of a tetrahedron with coterminus edges $\bar{a}, \bar{b}$, and $\bar{c}$ is $\frac{1}{6}[\bar{a}\ \bar{b}\ \bar{c}]$.

Hence, find the volume of tetrahedron whose coterminus edges are $\bar{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\bar{b} = -\hat{i} + \hat{j} + 2\hat{k}$ and $\bar{c} = 2\hat{i} + \hat{j} + 4\hat{k}$. (4)

Q. 30. Find the length of the perpendicular drawn from the point P (3, 2, 1) to the line

$$\bar{r} = (7\hat{i} + 7\hat{j} + 6\hat{k}) + \lambda(-2\hat{i} + 2\hat{j} + 3\hat{k})$$ (4)

Q. 31. If $y = \cos(m\cos^{-1} x)$ then show that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2 y = 0$$ (4)

Q. 32. Verify Lagrange's mean value theorem for the function $f(x) = \sqrt{x + 4}$ on the interval [0, 5]. (4)

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Q. 33. Evaluate :

$$\int \frac{2x^2 - 3}{(x^2 - 5)(x^2 + 4)} dx \qquad (4)$$

Q. 34. Prove that :

$$\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a - x)dx \qquad (4)$$

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