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2018 III 03

1100

J - 265

(E)

MATHEMATICS & STATISTICS (40) **(ARTS & SCIENCE)**

Time : 3 Hrs.

(7 Pages)

Max. Marks : 80

- Note :**
- (i) All questions are compulsory.
 - (ii) Figures to the right indicate full marks.
 - (iii) Graph of L.P.P. should be drawn on graph paper only.
 - (iv) Use of logarithmic table is allowed.
 - (v) Answers to the questions of Section - I and Section - II should be written in only one answer book.
 - (vi) Answer to every new question must be written on a new page.

SECTION – I

- Q. 1. (A)** Select and write the appropriate answer from the given alternatives in each of the following sub-questions : [12]
(6)

(i) If $A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$, then adjoint of matrix A is _____.

(a) $\begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 \\ 4 & -2 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$ (2)

(ii) The principal solutions of $\sec x = \frac{2}{\sqrt{3}}$ are _____.

(a) $\frac{\pi}{3}, \frac{11\pi}{6}$

(b) $\frac{\pi}{6}, \frac{11\pi}{6}$

(c) $\frac{\pi}{4}, \frac{11\pi}{4}$

(d) $\frac{\pi}{6}, \frac{11\pi}{4}$ (2)

(iii) The measure of acute angle between the lines whose direction ratios are 3, 2, 6 and -2, 1, 2 is _____.

(a) $\cos^{-1}\left(\frac{1}{7}\right)$

(b) $\cos^{-1}\left(\frac{8}{15}\right)$

(c) $\cos^{-1}\left(\frac{1}{3}\right)$

(d) $\cos^{-1}\left(\frac{8}{21}\right)$ (2)

(B) Attempt any THREE of the following : (6)

(i) Write the negations of the following statements :

(a) All students of this college live in the hostel.

(b) 6 is an even number or 36 is a perfect square. (2)

(ii) If a line makes angles α, β, γ with the co-ordinate axes, prove that $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + 1 = 0$. (2)

(iii) Find the distance of the point (1, 2, -1) from the plane $x - 2y + 4z - 10 = 0$. (2)

(iv) Find the vector equation of the line which passes through the point with position vector $4\hat{i} - \hat{j} + 2\hat{k}$ and is in the direction of $-2\hat{i} + \hat{j} + \hat{k}$. (2)

(v) If $\vec{a} = 3\hat{i} - 2\hat{j} + 7\hat{k}$, $\vec{b} = 5\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - \hat{k}$, then find $\vec{a} \cdot (\vec{b} \times \vec{c})$. (2)

Q. 2. (A) Attempt any TWO of the following : (6) [14]

(i) Using vector method prove that the medians of a triangle are concurrent. (3)

(ii) Using the truth table, prove the following logical equivalence :

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q). \quad (3)$$

(iii) If the origin is the centroid of the triangle whose vertices are $A(2, p, -3)$, $B(q, -2, 5)$ and $R(-5, 1, r)$, then find the values of p, q, r . (3)

(B) Attempt any TWO of the following : (8)

(i) Show that a homogeneous equation of degree two in x and y , i.e. $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through the origin if $h^2 - ab \geq 0$. (4)

(ii) In ΔABC , prove that $\tan\left(\frac{C-A}{2}\right) = \left(\frac{c-a}{c+a}\right) \cot \frac{B}{2}$. (4)

(iii) Find the inverse of the matrix, $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ using elementary row transformations. (4)

Q. 3. (A) Attempt any TWO of the following : (6) [14]

(i) Find the joint equation of the pair of lines passing through the origin, which are perpendicular to the lines represented by $5x^2 + 2xy - 3y^2 = 0$. (3)

- (ii) Find the angle between the lines $\frac{x-1}{4} = \frac{y-3}{1} = \frac{z}{8}$ and

$$\frac{x-2}{2} = \frac{y+1}{2} = \frac{z-4}{1} \quad (3)$$

- (iii) Write converse, inverse and contrapositive of the following conditional statement :

If an angle is a right angle then its measure is 90° . (3)

(B) Attempt any TWO of the following : (8)

(i) Prove that : $\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{56}{65}\right)$ (4)

- (ii) Find the vector equation of the plane passing through the points $A(1, 0, 1)$, $B(1, -1, 1)$ and $C(4, -3, 2)$. (4)

- (iii) Minimize $Z = 7x + y$ subject to
 $5x + y \geq 5$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$ (4)

SECTION - II

Q. 4. (A) Select and write the appropriate answer from the given [12]
alternatives in each of the following sub-questions : (6)

- (i) Let the p. m. f. of a random variable X be –

$$P(x) = \frac{3-x}{10} \text{ for } x = -1, 0, 1, 2$$

$$= 0 \quad \text{otherwise}$$

Then $E(X)$ is _____.

- (a) 1 (b) 2
(c) 0 (d) -1 (2)

(ii) If $\int_0^k \frac{1}{2+8x^2} dx = \frac{\pi}{16}$, then the value of k is _____.

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{4}$

(d) $\frac{1}{5}$

(2)

(iii) Integrating factor of linear differential equation

$x \frac{dy}{dx} + 2y = x^2 \log x$ is _____.

(a) $\frac{1}{x^2}$

(b) $\frac{1}{x}$

(c) x

(d) x^2

(2)

(B) Attempt any THREE of the following :

(6)

(i) Evaluate : $\int e^x \left[\frac{\cos x - \sin x}{\sin^2 x} \right] dx$

(2)

(ii) If $y = \tan^2(\log x^3)$, find $\frac{dy}{dx}$.

(2)

(iii) Find the area of ellipse $\frac{x^2}{1} + \frac{y^2}{4} = 1$.

(2)

(iv) Obtain the differential equation by eliminating the arbitrary constants from the following equation :

$y = c_1 e^{2x} + c_2 e^{-2x}$

(2)

(v) Given $X \sim B(n, p)$

If $n = 10$ and $p = 0.4$, find $E(X)$ and $\text{Var.}(X)$.

(2)

Q. 5. (A) Attempt any TWO of the following :

(6) [14]

(i) Evaluate : $\int \frac{1}{3 + 2 \sin x + \cos x} dx$ (3)

(ii) If $x = a \cos^3 t$, $y = a \sin^3 t$,

show that $\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$ (3)

(iii) Examine the continuity of the function :

$$f(x) = \frac{\log 100 + \log(0.01 + x)}{3x}, \text{ for } x \neq 0$$
$$= \frac{100}{3} \quad \text{for } x = 0; \text{ at } x = 0 \quad (3)$$

(B) Attempt any TWO of the following :

(8)

(i) Find the maximum and minimum value of the function :

$$f(x) = 2x^3 - 21x^2 + 36x - 20. \quad (4)$$

(ii) Prove that : $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$ (4)

(iii) Show that :

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx, \text{ if } f(x) \text{ is an even function.}$$

$$= 0, \quad \text{if } f(x) \text{ is an odd function.} \quad (4)$$

Q. 6. (A) Attempt any TWO of the following :

(6) [14]

(i) If $f(x) = \frac{x^2 - 9}{x - 3} + \alpha$, for $x > 3$

$= 5$, for $x = 3$

$= 2x^2 + 3x + \beta$, for $x < 3$

is continuous at $x = 3$, find α and β . (3)

(ii) Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{5x+1}{3-x-6x^2}\right)$ (3)

(iii) A fair coin is tossed 9 times. Find the probability that it shows head exactly 5 times. (3)

(B) Attempt any TWO of the following : (8)

(i) Verify Rolle's theorem for the following function :

$f(x) = x^2 - 4x + 10$ on $[0, 4]$ (4)

(ii) Find the particular solution of the differential equation : <https://www.maharashtrastudy.com>

$y(1 + \log x) \frac{dx}{dy} - x \log x = 0$

when $y = e^2$ and $x = e$. (4)

(iii) Find the variance and standard deviation of the random variable X whose probability distribution is given below :

x	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(4)

